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ON THE VIBRATIONS OF THREE-DIMENSIONAL ANGLED PIPING SYSTEMS CONVEYING FLUID

Chol Hui Pak*, Sung Chul Hong* and Young Suk Yun**

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The vibrations of three dimensional angled pipe systems conveying fluid are studied by using the finite element method. Exteneded Hamilton's principle is applied to derive the equations of motion. The characteristic matrices consisting of inertia, stiffness, and Coriolis terms are derived by variational method, in which the effects of the internal flow velocity and pressure are considered. The change of dynamic characteristics of the piping system due to the variation of flow velocity, pressure and the geometry of the system is investigated. As a result, it can be found that the natural frequency of the system decreases generally as the flow velocity and pressure increase and that the tendency is more significant as the geometry of the system is similar to the straight pipe.

Key Words: Piping System, Internal Flow, Static Instability, Critical Velocity, Natural Frequency

1. INTRODUCTION

The internal flow induced vibration problems occurs frequently in the fields of oil pipeline, nuclear reactor, and missile fuel lines. As the flow velocity and pressure change, the change of dynamic characteristics of these piping system have been interested, because the unwanted vibration and instability of the system may occur at a certain exciting frequency.

Some studies in this field are performed by finite element method. Mote(1971) studied the vibration and stability of cantilever pipe by FEM using Ritz method, Hill and Davis (1974) investigated the vibration of the pipe with constant curvature by FEM using Galerkin's method, and recently Kohli and Nakva(1984) analyzed the straight and curved tubes conveying fluid by means of straight beam finite elements.

In this paper, the piping systems which are constructed in 3 dimensional space are analyzed by FEM, using variational method, in which it is considered that the finite elements have total 12 degrees of freedom. The mass, Coriolis, and stiffness matrices are derived by variational method and static and dynamic analysis are performed. Through some examples, the relationship between natural frequencies and flow velocity is obtained. To verify the obtained result, the results obtained by presented method are compared with those by theoretical approach.

2. GENERAL THEORY

2.1 Model of Piping System

The piping system considered in this paper consists of n straight pipes joined together by n-1 rigid elbows, and the both ends are clamped, as shown in Fig. 1. The elbow parts are idealized as a very short massless rigid part in the viewpoint that the length of the elbows is very short when it is compared with that of straight pipes. The piping system is conveying the fluid of velocity c and mass per unit length m_f.

In this analysis, the effects of gravity, shear deformation and rotary inertia are neglected. And it is assumed that the flow of fluid is constant, pressure drop is negligible, and the deflections are small.

2.2 Equations of Motion of Finite Elements

The piping system may be represented by a sum of the n dividual finite elements. The finite element in local coordinates is shown in Fig. 2.

In Fig.2, j and k represent the node points of finite element (e). The each node point has 6 degrees of freedom which consist of 3 linear displacements x_m, y_m, z_m and 3 rotational displacements θ_x, θ_y, θ_z. Therefore the finite element (e) has the total 12 degrees of freedom and the displacement vector $\{\delta_e\}$ may be represented by

$$\{\delta_e\}=\{\delta_{jx},\delta_{jy},\delta_{jz},\theta_{jx},\theta_{jy},\theta_{jz},\delta_{kz},\delta_{ky},\delta_{kz},\theta_{kx},\theta_{ky},\theta_{kz}\}^T \tag{1}$$

The equations of motion of the finite element (e) may be derived by using the extended Hamilton's principle

$$\int_{t1}^{t2}\{\delta T_p-\delta V_p+\delta T_f+\delta W_{pre}-\delta W_{pn}\}dt=0 \tag{2}$$

where T_p is kinetic energy of the pipe, V_p a potential energy of the pipe, T_f a kinetic energy of the fluid, W_{pre} is a work of pipe done by pressure, W_{pn} a potential energy due to nodal forces, respectively. The corresponding terms are represented in the form

$$\delta T_p=\delta[\int_0^{Le}\frac{1}{2}\{m_p\left(\frac{\partial u}{\partial t}\right)^2+m_p\left(\frac{\partial v}{\partial t}\right)^2+m_p\left(\frac{\partial w}{\partial t}\right)^2\}\mathrm{dx}] \tag{3}$$

$$\delta V_p=\delta[\int_0^{Le}\frac{1}{2}\{EA_p\left(\frac{\partial u}{\partial x}\right)^2+EI^z\left(\frac{\partial^2 v}{\partial x^2}\right)^2+EI^y\left(\frac{\partial^2 w}{\partial x^2}\right)^2$$

*Department of Mechanical Engineering, Inha University, 253, Yonghyun-Dong, Nam-ku, Inchon, 420-751, Korea

**Department of Mechanical Engineering, Yosu National Fisheries College, 195, Kuk-Dong, Yosu, Chonam, 550-749, Korea

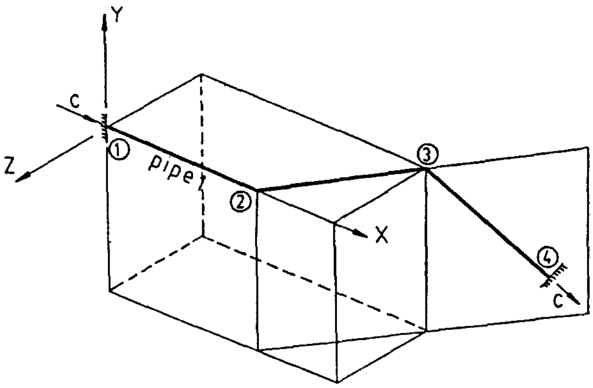

Fig. 1 The model of the system

$$+GJ\left(\frac{\partial \phi}{\partial x}\right)^2+P_{xe}\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]\}]$$

$$\delta T_f=\delta\left[\int_0^{Le}\frac{1}{2}m_f\{c^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial x}+c\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial t}+c\frac{\partial w}{\partial x}\right)^2\}dx\right]$$

$$\delta W_{pre}=\delta\left[\int_0^{Le}\frac{1}{2}pA\{\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\}dx\right]$$

$$\begin{aligned}\delta W_{pn}=&-\delta[P_{jx}u(o)+P_{kx}u(L_e)+P_{jy}v(o)\\&+P_{ky}v(L_e)+P_{jx}w(o)+P_{kz}w(L_e)\\&+M_{jx}\phi(o)+M_{kx}\phi(L_e)+M_{jx}\frac{\partial v(o)}{\partial x}\\&+M_{kx}\frac{\partial v(L_e)}{\partial x}+M_{jy}\frac{\partial w(o)}{\partial x}+M_{ky}\frac{\partial w(L_e)}{\partial x}]\end{aligned}$$

where c is a fluid velocity, m_p a pipe mass per unit pipe length, m_f a fluid mass per unit pipe length, p a fluid pressure, J_o a tortional inertia, A an internal area of the pipe, A_p an area of the pipe, EI^y and EI^z stiffness coefficients, GJ a tortional inertia coefficient and P_{xe} an initial tension force in the finite element (e) of the pipe. And the coordinates $u,v,w,$ and ϕ represent a logitudinal displacement, a bending displacement in xy plane, a bending displacement in zx plane, and a rotational displacement with respect to the longitudinal axis, respectively.

Let's define the field variables of the form

$$\begin{matrix}u(x,\ t)=[N_1(x)]\{U_e(t)\} & v(x,\ t)=[N_2(x)]\{V_e(t)\}\\ w(x,t)=[N_3(x)]\{W_e(t)\} & \phi(x,\ t)=[N_1(x)]\{\phi_e(t)\}\end{matrix}\tag{4}$$

where the displacement vectors of finite element (e) are

$$\begin{aligned}&\{U_e(t)\}=\{\delta_{jx},\ \delta_{kx}\}^T\\&\{V_e(t)\}=\{\delta_{jy},\ \theta_{jz},\ \delta_{ky},\ \theta_{kz}\}^T\\&\{W_e(t)\}=\{\delta_{jz},\ \theta_{jy},\ \delta_{kz},\ \theta_{ky}\}^T\\&\{\phi_e(t)\}=\{\theta_{jx},\ \theta_{kx}\}^T\end{aligned}\tag{5}$$

and the interpolation functions $[N_i(x)]\ (i=1,2,3)$ are

$$\begin{aligned}&[N_1(x)]=[1-\frac{x}{L_e},\ \frac{x}{L_e}]\\&[N_2(x)]=[1-\frac{3x^2}{L_e^2}+\frac{2x^3}{L_e^3},\ x-\frac{2x^2}{L_e}+\frac{x^3}{L_e^2},\\&\qquad\frac{3x^2}{L_e^2}-\frac{2x^3}{L_e^3},\ -\frac{x^2}{L_e}+\frac{x^3}{L_e^2}]\end{aligned}\tag{6}$$

$$\begin{aligned}&[N_3(x)]=[1-\frac{3x^2}{L_e^3}+\frac{2x^3}{L_e^3},\ -\left(x-\frac{2x^2}{L_e}+\frac{x^3}{L_e^2}\right),\\&\qquad\frac{3x^2}{L_e^2}-\frac{2x^3}{L_e^3},\ -\left(-\frac{x^2}{L_e}+\frac{x^3}{L_e^2}\right)]\end{aligned}\tag{6}$$

By inserting the Eq. (4) into the Eq.(2), we can obtain the equations of motion of a finite element (e) in local coordinate

$$\underset{12\times12}{[M]_e}\underset{12\times1}{\{\ddot{\delta}_e\}}+\underset{12\times12}{[D]_e}\underset{12\times1}{\{\dot{\delta}_e\}}+\underset{12\times12}{[K]_e}\underset{12\times1}{\{\delta_e\}}=\underset{12\times1}{\{P_e\}}\tag{7}$$

where $\{P_e\}$ are nodal forces of the finite element (e) of the form

$$\{P_e\}=\{P_{jx},P_{jy},P_{jz},M_{jx},M_z,M_{ju},P_{kx},P_{ky},P_{kz},M_{kx},M_{ky},M_{kz}\}^T\tag{8}$$

and the mass, Coriolis and stiffness matrices are as follows.

$$[K]_e=\frac{EI}{L_e^3}\left[\begin{array}{cccccccccccc}\frac{AL_e^2}{I}&0&0&0&0&0&-\frac{AL_e^2}{I}&0&0&0&0&0\\ &(12-\frac{1.2V_f}{L_e})&0&0&0&(6L_e-\frac{V_f}{10})&0&(-12+\frac{1.2V_f}{L_e})&0&0&0&(6L_e-\frac{V_f}{10})\\ &&(12-\frac{1.2V_f}{L_e})&0&-(6L_e-\frac{V_f}{10})&0&0&0&(-12+\frac{1.2V_f}{L_e})&0&-(6L_e-\frac{V_f}{10})&0\\ &&&\frac{2L_e^2G}{E}&0&0&0&0&0&-\frac{2L_e^2G}{E}&0&0\\ &&&&(4L_e^2-\frac{2L_eV_f}{15})&0&0&0&-(6L_e+\frac{V_f}{10})&0&(2L_e^2+\frac{L_eV_f}{30})&0\\ &&&&&(4L_e^2-\frac{2L_eV_f}{15})&0&(-6L_e+\frac{V_f}{10})&0&0&0&(2L_e^2+\frac{L_eV_f}{30})\\ &&\text{Symmetric}&&&&\frac{AL_e^2}{I}&0&0&0&0&0\\ &&&&&&&(12-\frac{1.2V_f}{L_e})&0&0&0&(-6L_e+\frac{V_f}{10})\\ &&&&&&&&(12-\frac{1.2V_f}{L_e}&0&-(-6L_e+\frac{V_f}{10})&0\\ &&&&&&&&&\frac{2L_e^2G}{E}&0&0\\ &&&&&&&&&&(4L_e^2-\frac{2L_eV_f}{15})&0\\ &&&&&&&&&&&(4L_e^2-\frac{2L_eV_f}{15})\end{array}\right]$$

where $V_f=(m_fc^2+pA-P_{xe})L_e^3/EI$

$I=I^z=I^y=\frac{J}{2}$

$$
[M]_e=\frac{(m_p+m_f)L_e}{420}\left[\begin{array}{cccccccccccc}
140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\
 & 156 & 0 & 0 & 0 & 22L_e & 0 & 54 & 0 & 0 & 0 & -13L_e \\
 & & 156 & 0 & -22L_e & 0 & 0 & 0 & 54 & 0 & 13L_e & 0 \\
 & & & \frac{140I}{A} & 0 & 0 & 0 & 0 & 0 & \frac{70I}{A} & 0 & 0 \\
 & & & & 4L_e^2 & 0 & 0 & 0 & -13L_e & 0 & -3L_e^2 & 0 \\
 & & & & & 4L_e^2 & 0 & 13L_e & 0 & 0 & 0 & -3L_e^2 \\
 & & & & & & 140 & 0 & 0 & 0 & 0 & 0 \\
 & & \text{Symmetric} & & & & & 156 & 0 & 0 & 0 & -22L_e \\
 & & & & & & & & 156 & 0 & 22L_e & 0 \\
 & & & & & & & & & \frac{140I}{A} & 0 & 0 \\
 & & & & & & & & & & 4L_e^2 & 0 \\
 & & & & & & & & & & & 4L_e^2
\end{array}\right]
$$

$$
[D]_e=\frac{m_f c}{30}\left[\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 6L_e & 0 & 30 & 0 & 0 & 0 & -6L_e \\
 & & 0 & 0 & -6L_e & 0 & 0 & 0 & 30 & 0 & 6L_e & 0 \\
 & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & -6L_e & 0 & -L_e^2 & 0 \\
 & & & & & 0 & 0 & 6L_e & 0 & 0 & 0 & -L_e^2 \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & \text{Skew-symmetic} & & & & & 0 & 0 & 0 & 0 & 6L_e \\
 & & & & & & & & 0 & 0 & -6L_e & 0 \\
 & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & 0
\end{array}\right]
$$

2.3 Equations of motion of the system

Equation (7) gives expression of equations of motion of a finite element (e) in local coordinates. Therefore it is necessary to transform the coordinate to represent equations of motion in global coordinate given by

$$\underset{12\times1}{\{\bar{\delta}_e\}}=\underset{12\times12}{[T]^{-1}}\underset{12\times1}{\{\delta_e\}}=[T]^T\{\delta_e\} \qquad \{\bar{P}_e\}=[T]^T\{P_e\}$$

$$[\bar{K}]_e=[T]^T[K]_e[T]$$
$$[\bar{M}]_e=[T]^T[K]_e[T]$$
$$[\bar{D}]_e=[T]^T[D]_e[T]$$

The superposition of all transformed finite element matrices $[\bar{M}]_e$, $[\bar{K}]_e$, $[\bar{D}]_e$ results in the equations of motion of the total system

$$[\bar{M}]\{\ddot{\bar{\delta}}\}+[\bar{D}]\{\dot{\bar{\delta}}\}+[\bar{K}]\{\bar{\delta}\}=\{\bar{P}\} \tag{9}$$

where

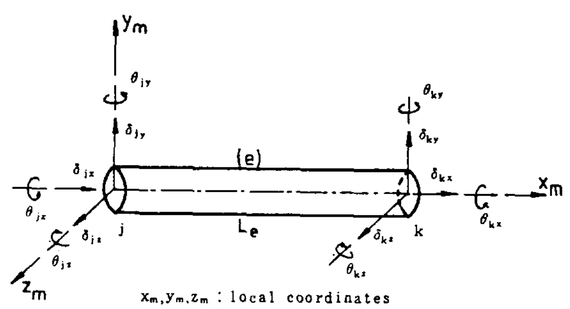

Fig. 2 Finite element model degree of freedom

$$[\bar{M}]=\sum_{e=1}^{n}[\bar{M}]_e, \text{ symmetric } (6n+6)\times(6n+6)$$
$$[\bar{D}]=\sum_{e=1}^{n}[\bar{D}]_e, \text{ skew-symmetric } (6n+6)\times(6n+6)$$
$$[\bar{K}]=\sum_{e=1}^{n}[\bar{K}]_e, \text{ symmetric } (6n+6)\times(6n+6) \tag{10}$$
$$\{\bar{\delta}\}=\{\bar{\delta}\}_s+\{\bar{\delta}\}_d, \ (6n+6)\times(1)$$
$$\{\bar{P}\}=\{\bar{P}\}_s+\{\bar{P}\}_d, \ (6n+6)\times(1)$$

where n is a number of element, subscripts s and d represent static and dynamic quantities, respectively. The Eq. (9) consists of the static equilibrium equations and dynamic equations. To separate the static equilibrium equations and the dynamic equations in Eq. (9), insert Eq. (10) into Eq. (9), then we can obtain the static equilibrium equation

$$[\bar{K}]\{\bar{\delta}\}_s=\{\bar{P}\}_s \tag{11}$$

and dynamic equations

$$[\bar{M}]\{\ddot{\bar{\delta}}\}_d+[\bar{D}]\{\dot{\bar{\delta}}\}_d+[\bar{K}]\{\bar{\delta}\}_d=\{\bar{P}\}_d=\{0\} \tag{12}$$

2.4 Static Analysis

In Eq. (11), the stiffness matrix $[\bar{K}]$ is represented by

$$[\bar{K}]=\sum_{e=1}^{n}\left[[\bar{K}]_{e,E}-(m_f c^2+pA-P_{xe})[\bar{K}]_{e,o}\right]$$

where $[\bar{K}]_{e,E}$ is a matrix derived from the elastic characteristics of the pipe and $[\bar{K}]_{e,o}$ is a matrix derived from the centrifugal force and pressure of fluid and the initial tension force. The initial tension force is a static force applied to the system when the system is in the static equilibrium state and given by

$$P_{xe}=\frac{EA_p}{L_e}(\delta_{kx}-\delta_{jx})$$

The displacements and forces at the static equilibrium state are determined by inserting the boundary conditions and the resultant forces at elbow parts. The resultant forces occur due to the momentum changes and pressure when the fluid passes by the elbow parts. The forces act in the plane constituted by two neighboring straight pipes and the magnitudes are determined by the fluid velocity and pressure and the curved angles. As an example, when the geometry of the piping system has form shown in Fig. 3, the nodal forces P_2 and P_3 are

$$\{\bar{P}_2\}_s=\begin{bmatrix}\bar{P}_{2x}\\ \bar{P}_{2y}\\ \bar{P}_{2z}\\ \bar{M}_{2x}\\ \bar{M}_{2y}\\ \bar{M}_{2z}\end{bmatrix}=R_1\begin{bmatrix}1+\cos\alpha_1\\ 0\\ \sin\alpha_1\\ 0\\ 0\\ 0\end{bmatrix},$$

$$\{\bar{P}_3\}_s=\begin{bmatrix}\bar{P}_{3x}\\ \bar{P}_{3y}\\ \bar{P}_{3z}\\ \bar{M}_{3x}\\ \bar{M}_{3y}\\ \bar{M}_{3z}\end{bmatrix}=R_2\begin{bmatrix}-\cos\alpha_1(1+\cos\alpha_2)\\ \sin\alpha_2\\ -\sin\alpha_1(1+\cos\alpha_2)\\ 0\\ 0\\ 0\end{bmatrix}$$

where

$$R_i=(m_f c^2+pA)(1+\cos\alpha_i),\quad i=1,2.$$

2.5 Dynamic Analysis

After inserting boundary conditions into Eq. (12), we can obtain the equation

$$[\tilde{M}]\{\ddot{\tilde{\delta}}\}_d+[\tilde{D}]\{\dot{\tilde{\delta}}\}_d+[\tilde{K}]\{\tilde{\delta}\}_d=\{0\} \tag{13}$$

Since this Eq. (13) is an equation of free vibration of a conservative gyroscopic system, the natural frequencies are real value (Meirovitch, 1980). To solve the Eq. (13), let

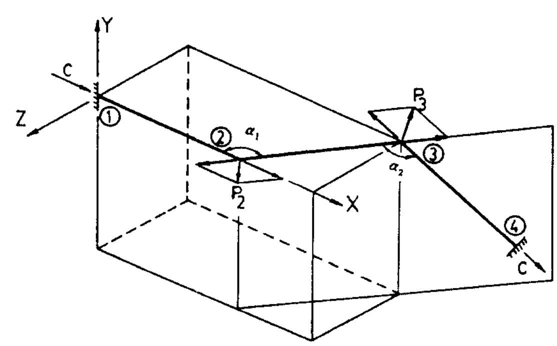

Fig. 3 The equivalent resultant forces acting on the angled parts

$$\{\dot{\tilde{\delta}}\}=\{\tilde{\xi}\}$$

then Eq. (13) becomes

$$\{\dot{Z}\}=[S]\{Z\} \tag{14}$$

where

$$\{Z\}=\begin{Bmatrix}\tilde{\delta}\\ \tilde{\xi}\end{Bmatrix},\quad [S]=\begin{bmatrix}[O] & [I]\\ -[\tilde{M}]^{-1}[\tilde{K}] & -[\tilde{M}]^{-1}[\tilde{D}]\end{bmatrix}$$

therefore we can obtain the natrual frequencies and mode shapes by solving the characteristic equation of

$$(\lambda[I]-[S])\{a\}=0 \tag{15}$$

On the other hand, when the mass ratio $\frac{m_f}{m_p+m_f}$ is small, the skew-symmetric matrix $[\tilde{D}]$ does not affect largely on the natural frequencies (Chen, 1972 ; Chen, 1973 ; Hill and Davis, 1974). This leads to the relation

$$[\tilde{M}]\{\delta\}_d+[\tilde{K}]\{\tilde{\delta}\}_d=\{0\}$$

Therefore,, we can obtain the natural frequencies and mode shapes by solving the characteristic equation of

$$([\tilde{K}]-\omega^2[\tilde{M}])\{a\}=0 \tag{16}$$

3. NUMERICAL EXAMPLES

By some examples, the dynamic characteristics of piping system are investigated. The flowchart for computation is shown in Fig.4. As an example, the piping system shown in

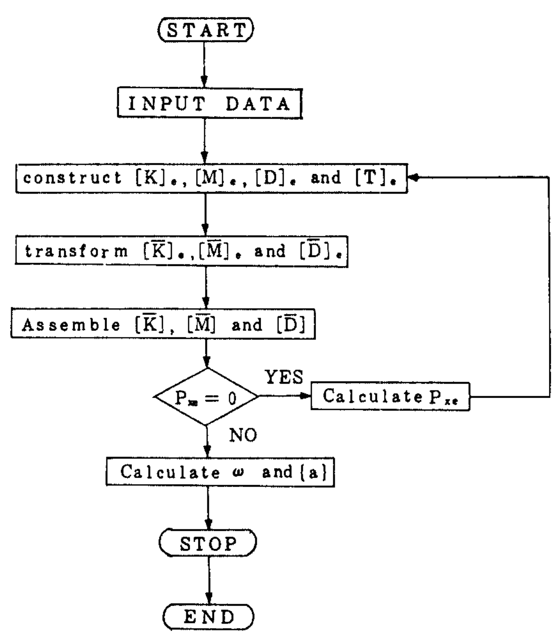

Fig. 4 Flow-chart of the computer program for the pipe system conveying fluid including initial tension forces

Table 1 Material properties of copper pipe

E	$124.110\times10^{9}(N/m^2)$	J	$4.237\times10^{-9}(m^4)$
G	$46.197\times10^{9}(N/m^2)$	J_o	$3.790\times10^{-9}(Kg/m^2)$
d_o	$19.050\times10^{-3}(m)$	m_f	0.234 (Kg/m)
d_i	$17.250\times10^{-3}(m)$	m_p	0.459 (Kg/m)
I^y, I^z	$2.1180\times10^{-9}(m^4)$	p	0.0 (N/m^2)

Table 2 global coordinate values of the nodal points

Node point	X	Y	Z
1	O	O	O
2	L_e	O	O
3	$2L_e$	O	O
4	$2L_e - L_e \cos\alpha_1$	O	$-L_e \sin\alpha_1$
5	$2L_e - 2L_e \cos\alpha_1$	O	$-2L_e \sin\alpha_1$
6	$2L_e - 2L_e \cos\alpha_1$ $+L_e \cos\alpha_1 \cos\alpha_2$	$-L_e \sin\alpha_2$	$-2L_e \sin\alpha_1$ $+ L_e \sin\alpha_1 \cos\alpha_2$
7	$2L_e - 2L_e \cos\alpha_1$ $+2L_e \cos\alpha_1 \cos\alpha_2$	$-2L_e \sin\alpha_2$	$-2L_e \sin\alpha_1$ $+2L_e \sin\alpha_1 \cos\alpha_2$

Fig.3 is considered.

3.1 Input Data

The piping system is made of copper pipes which have the material properties and dimensions tabulated in Table 1.
The positions of node points in global coordinates are given in Table 2.
The output is represented by means of dimensionless frequency

$$\Omega=\left[\frac{m_p+m_f}{EI}\right]^{1/2}L^2\omega$$

and dimensionless velocity

$$C=\left[\frac{m_f}{EI}\right]^{1/2}Lc$$

3.2 Examples and Results

Firstly, to verify the obtained results, the change of natural frequency of piping system which consists of two straight pipe and one elbow, with the angle 135° according to flow velocity change is obtained by presented method and theoretical approach (Hong, 1987), and plotted in Fig.5. There are good agreements between them. As the flow velocity and pressure increases, the natural frequency decreases and the static instability buckling phenomenon occurs at critical velocity. The relations between frequencies and velocity with respect to the change of geometry of the system are found and plotted in Fig.6,7,8. We can find the fact that the critical velocity is infinite when the angles of the piping system are all 90° from Fig.6. However, when the angle of the piping system is not 90°, the natural frequencies are reduced as the flow velocity increases, and the critical velocity exists.

Blevins(1977) derived the equation

$$\Omega(C)=\Omega(0)\left[1-\frac{C^2}{C_{cr}^2}\right]^{1/2} \tag{17}$$

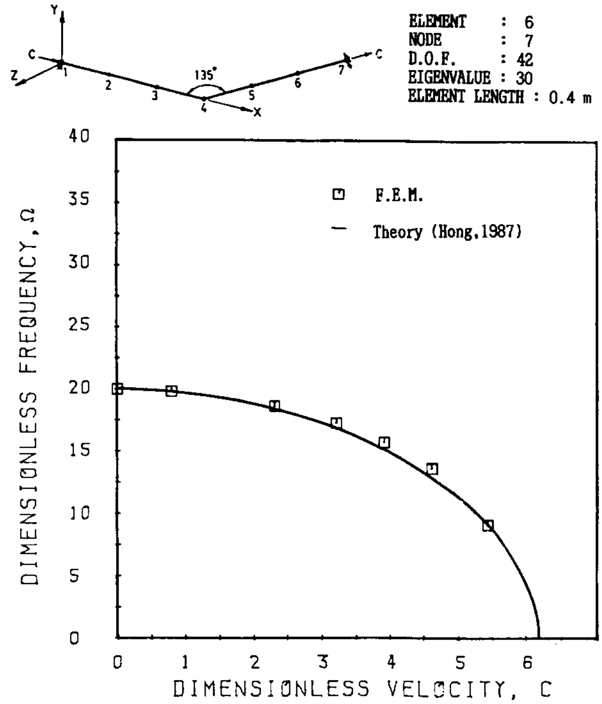

Fig. 5 Comparison of theorical result (Hong, 1987) and F.E.M. result of fundamental frequency for the angle of 135°

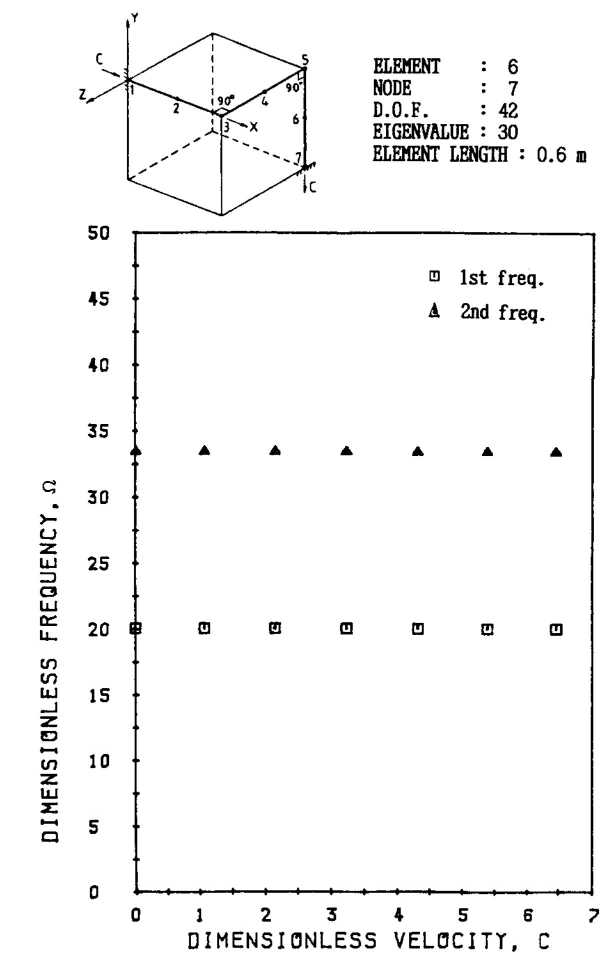

Fig. 6 The fundamental and second frequency of vibrations of the angled pipe conveying fluid for the angle of 90°

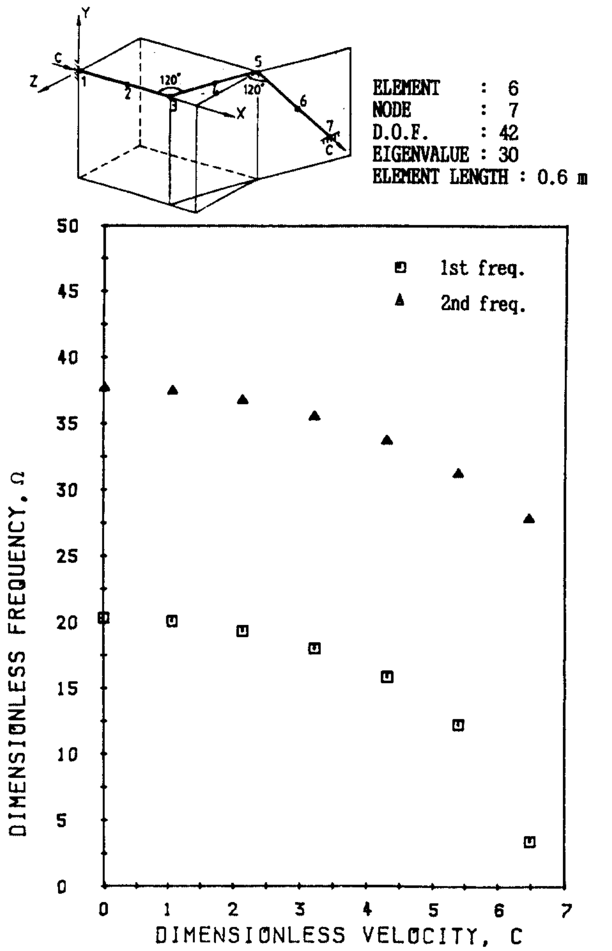

Fig. 7 The fundamental and second frequency of vibrations of the angled pipe conveying fluid for the angles of 120°

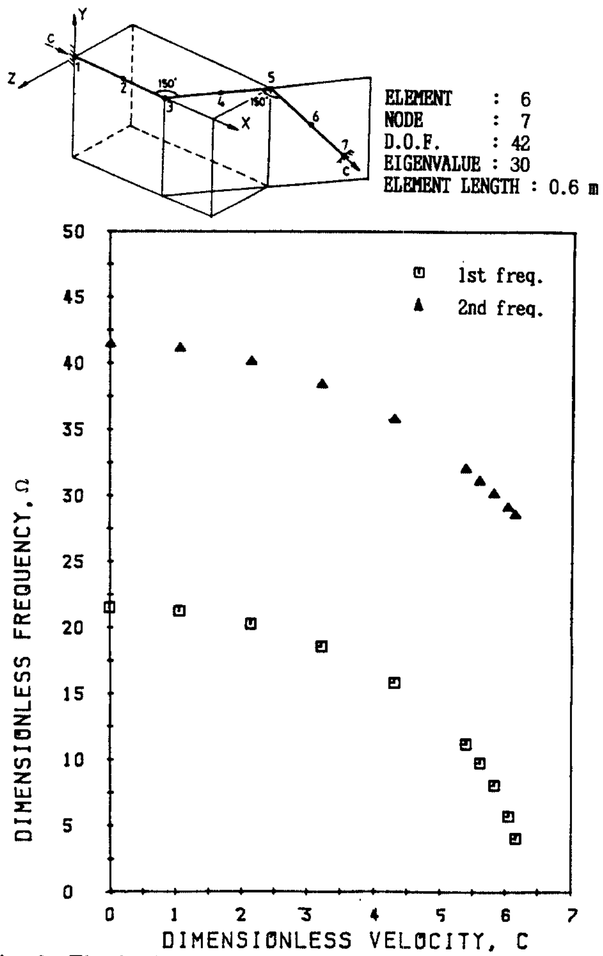

Fig. 8 The fundamental and second frequency of vibrations of the angled pipe conveying fluid for the angles of 150

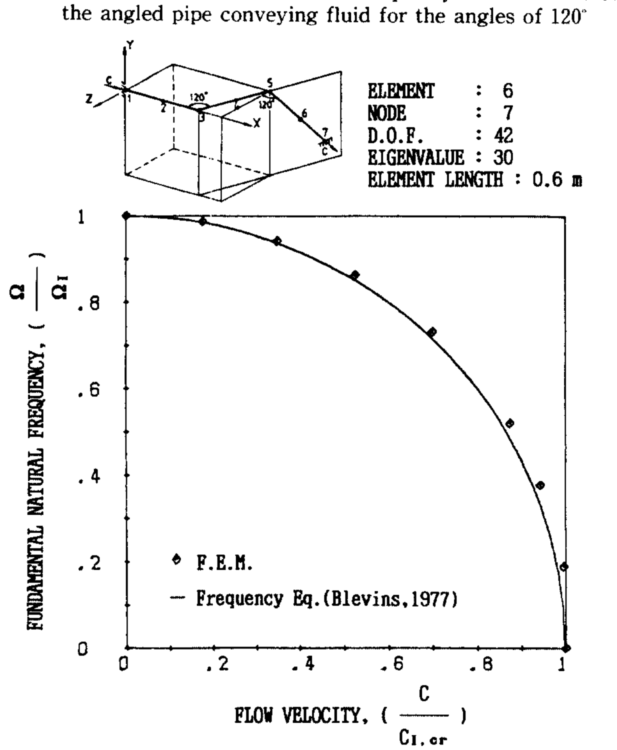

Fig. 9 Comparison of Blevins(1977) and F.E.M. result of fundamental frequency as a function of flow velocity for the angles of 120°

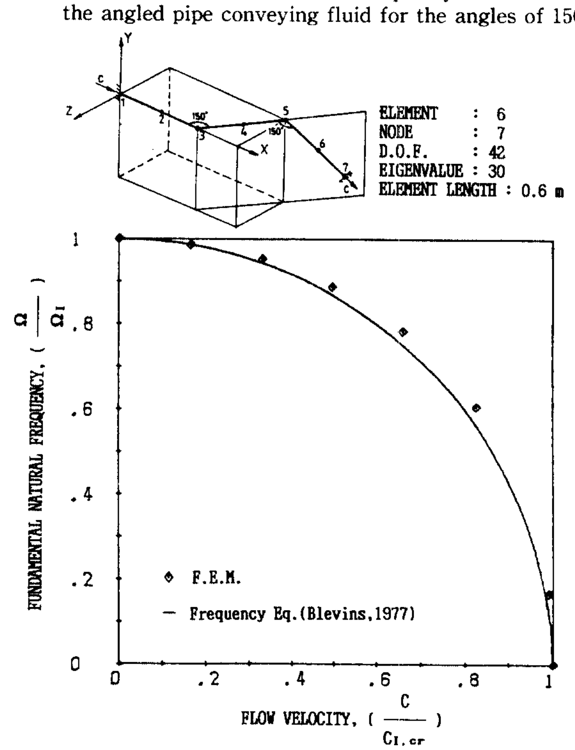

Fig. 10 Comparison of Blevins(1977) and F.E.M. result of fundamental frequency as a function of flow velocity for the angles of 150°

which denotes the relations betweens frequency and velocity of a system with only a straight pipe. It can be shown that the results obtained from the frequency equation are good approximate relation, as shown in Fig.9 and Fig.10. It may be considered in the viewpoint of the designer's concept that the frequency equation can be used for approximate analysis of the system's dynamic characteristics with good accuracy within 2% error in the range $C<0.5C_{cr}$ and 10% error in the range $C<0.9C_{cr}$, even if the systems do not consist of only a straight pipe.

4. CONCLUSIONS

The vibration of three dimensional angled piping systems is analyzed by the finite element method. As a result, we can obtain the following conclusions :

(1) The vibration analysis for investigating the effects of flow velocity and pressure on dynamic characteristics of the system is performed.

(2) When the angle of curved part is obtuse, as the flow velocity or pressure increases, the fundamental frequency decreases, and the critical velocity or pressure become infinite, therefore the buckling phenomena do not occur.

(3) If the angles are obtuse, as the angles approach to 180°, the buckling phenomena occur at the relatively low flow velocity or pressure.

(4) Through the investigation of the relationship for flow velocity versus natural frequencies, it can be shown that the approximate equation

$$\Omega(C)=\Omega(0)\left[1-\frac{C^2}{C_{cr}^2}\right]^{1/2}$$

(where $\Omega(0)$ = natural frequency when the velocity is zero, C = flow velocity, C_{cr} = critical velocity of the system) can be used with 10% error within the range of flow velocity $C<0.9C_{cr}$.

(5) The developed computer program may be used to analyze the vibration of piping structure with the other general purpose program in which the other effects for piping systems are considered.

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